"On a Heuristic Point of View Concerning the Production & Transformation of Light"

Annalen der Physik 17 (1905), 132-48

Dated 17 march 1905 Received 18 march 1905

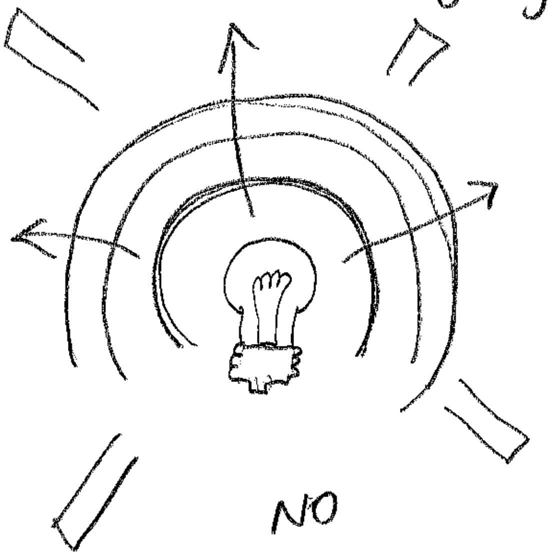
Before doctoral dissertation and Brownian motion papers!

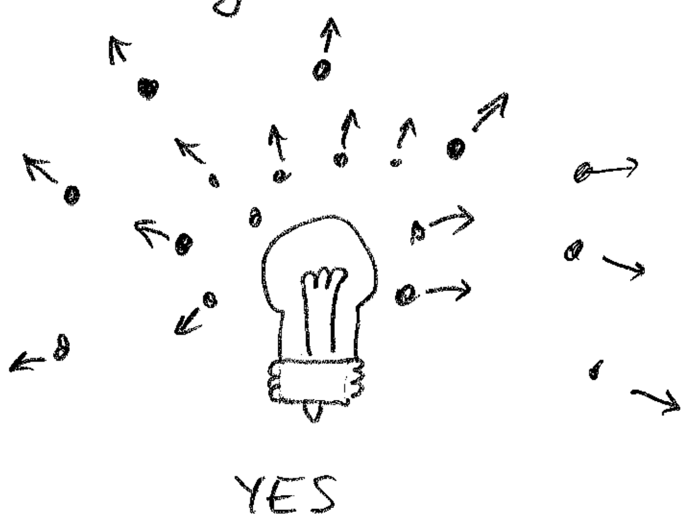
Notes by John D. Norton

Principal claim

Light quantum hypothesis (sloppy version, introduction)

"... in the propagation of a light ray emitted from a point source, the energy is not distributed continuously over ever-increasing volumes of space, but consists of a finite number of energy quanta localized at points of space that move without dividing, and can be absorbed or generated only as complete units."





Light quantum hypothesis. (Precise version, §6)

"... monochromatic radiation of low density (with the range of validity of Wien's radiation formula) behaves thermodynamically as if it consisted of mutually independent energy quanta of magnitude $\frac{R\beta\nu}{N}$."

} Large ν/T

In 1905, just means "units"

By context they are localized in space

$h\nu$ since

$\frac{R}{N} = k$

$\beta = h/k$

"seems reasonable to investigate"

"whether laws governing the emission and transformation of light are also constructed as if light consisted of such energy quanta"

No assertion made concerning momentum. Comes later

[Is the hypothesis literally true or (just) heuristically useful?]

Structure of Paper

Introduction. Discreteness & continuity of energy

§1 On a difficulty concerning the theory of "Black body radiation"

- Clean statement (not found in Planck) that observations contradict classical analysis.
- Derivation of classical [Rayleigh-Jeans] distribution
- Impossibility of equilibrium

§2 On Planck's Determination of Elementary Quanta

Planck's formula of 1901 reverts to classical formula §1 for high energy density and long wavelengths. Fix constants.

§3 On the Entropy of Radiation

recapitulates known theorem that will allow passage (observed energy) → (entropy)

§4 Limiting Law for the Entropy of Monochromatic Radiation at Low Radiation Density

Entropy $S \propto \log(\text{volume } v)$

§5 Molecular-Theoretical Investigation of the Dependence on Volume of the Entropy of Gases and Dilute Solutions

core argument

$S \propto \log v$ ⟷ spatially localized, independent units of energy

§6 Interpretation According to Boltzmann's Principle ...

core argument

Deduce light quantum hypothesis

§7 On Stokes's Rule

§8 On the Generation of Cathode Rays by Illumination of Solid Bodies

§9 On the Ionization of Gases by Ultraviolet Light

Experiments on production & transformation of light that support light quantum hypothesis.

Introduction

Received view

ordinary matter (atoms, electrons)
$\underbrace{\hphantom{ordinary matter}}$
energy distributed discretely

vs

Radiation (maxwell's theory)
$\underbrace{\hphantom{Radiation theory}}$
energy distributed continuously.

↑

- supported by experiments on "diffraction, reflection, refraction, dispersion, etc. ..." BUT
- "Black body radiation, photoluminesce, production of cathode rays by ultraviolet light ..." suggest <u>dis</u>continuous distribution

compatibility? These "refer to time averages rather than instantaneous values"

"Revolutionary"

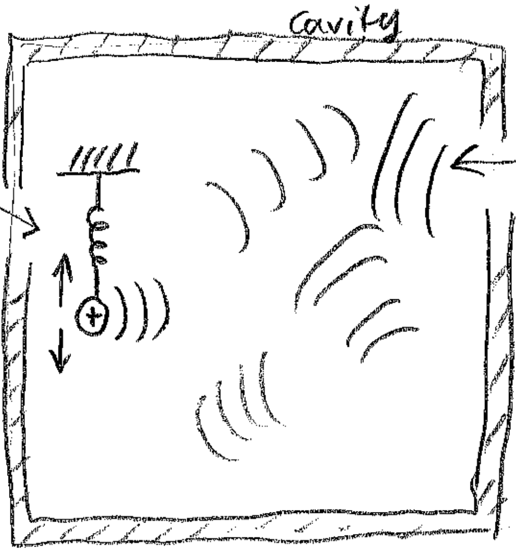
Einstein to Habicht, 1905

overturns <u>the</u> most successful theory of 19th c.

↑

special relativity <u>perfects</u> maxwell's electrodynamics.

6| On a difficulty concerning the theory of black body radiation



Electric resonators in equilibrium with cavity (black body) radiation.

cavity (black body) radiation at temperature T

ρ_ν = energy density at frequency ν

$\Uparrow$

Same as Rayleigh-Jeans law (AE does not say this!)

$$\rho_\nu = k\,\frac{8\pi\nu^2}{c^3}\,T$$

R/N

Mean energy for linear motion in one direction

$$\bar{E}_\nu = kT$$

resonant frequency

R/N

Planck's condition for equilibrium

$$\Leftarrow \quad \bar{E}_\nu = \frac{L^3}{8\pi\nu^3}\,\rho_\nu \quad \Rightarrow$$

(L = c)

$\Uparrow$ Equipartition Theorem
Two degrees freedom

Resonator

$$H = \frac{1}{2}Kx^2 + \frac{1}{2m}p^2$$

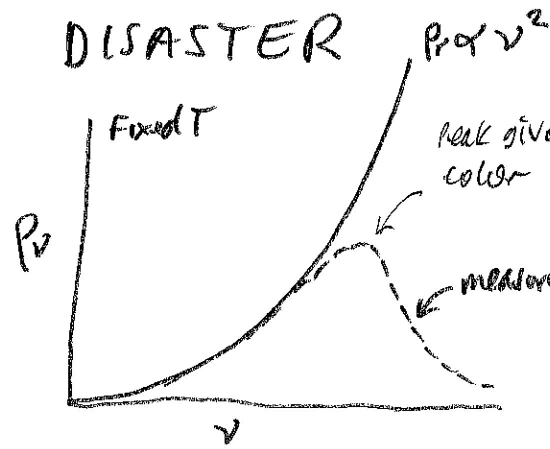
energy, position, spring const., mass, momentum

$\Downarrow$

DISASTER

$\rho_\nu \propto \nu^2$

1. Violates measured density



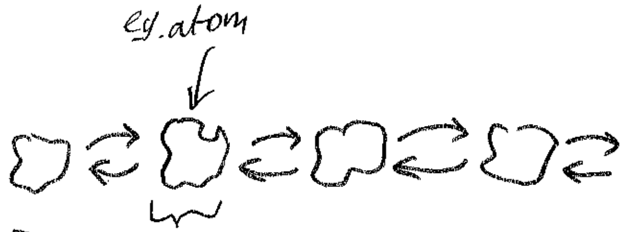
2. Total energy $\int_0^\infty \rho_\nu d\nu = \infty$

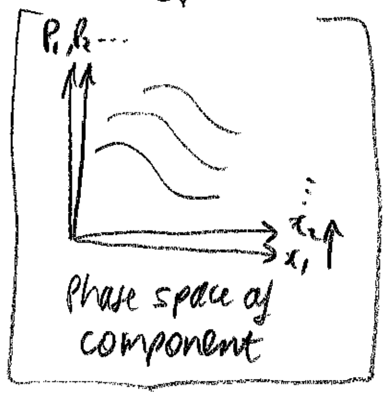
... No equilibrium with matter possible

JDN Heat capacity radiation also infinite!

Equipartition theorem

IF ① component in equilibrium with many others at T





Probability density

energy

$$p = \exp\left(-E/kT\right)$$

② Energy E is quadratic in phase space coordinates

$$E = \underbrace{c_1 x_1^2 + c_2 x_2^2 + \cdots d_1 p_1^2 + d_2 p_2^2 + \cdots}_{n \text{ terms} \Rightarrow \text{"}n \text{ degrees freedom"}}$$

c_i, d_i constants

THEN mean energy of component

$$\bar{E} = \int E\, p\, dx_1 \cdots = \underbrace{n\left(\tfrac{1}{2}kT\right)}_{\text{"}\frac{1}{2}kT \text{ per degree freedom}}$$

by standard, tedious integral

e.g. monatomic gas

$$E = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(v_x^2 + v_y^2 + v_z^2) = \frac{1}{2m}\left(\underbrace{p_x^2 + p_y^2 + p_z^2}_{3 \text{ degrees freedom}}\right)$$

$$\therefore \quad \bar{E} = 3\left(\tfrac{1}{2}kT\right) = \tfrac{3}{2}kT$$

$$\boxed{\begin{array}{ll}\text{IF} & \text{THEN}\\ E = bx^n + \underbrace{E'(y,z,\ldots)}_{m-1\text{ variables}} & \bar{E} = \frac{1}{n}kT + \bar{E}'\\ \frac{d^m P}{dx\,dy\ldots} = \exp(-E/kT) & \end{array}}$$

mean energy if $E = E'$ only (defined)

$$\bar{E} = \frac{\int E e^{-E/kT}\,dx\,dy\ldots}{\int e^{-E/kT}\,dx\,dy\ldots} = \underbrace{\frac{\int bx^n e^{-E/kT}\,dx\,dy\ldots}{\int e^{-E/kT}}} + \underbrace{\frac{\int E' e^{-E/kT}}{\int e^{-E/kT}}}$$

analogously

$$\frac{\int bx^n e^{-\frac{bx^n}{kT}}\,dx \cancel{\int e^{-E'/kT}\,dy\,dz\ldots}}{\int e^{-\frac{bx^n}{kT}}\,dx \cancel{\int e^{-E'/kT}\,dy\,dz}} \qquad \frac{\int E' e^{-E'/kT}\,dy\,dz\ldots}{\int e^{-E'/kT}\,dy\,dz\ldots}$$

$$= \frac{\int bx^n e^{-bx^n/kT}\,dx}{\int e^{-bx^n/kT}\,dx} \qquad = \bar{E}'$$

$$= -\frac{\partial}{\partial(1/kT)} \ln \int e^{-bx^n/kT}\,dx = -\frac{\partial}{\partial(1/kT)} \ln \int e^{-b\left(\frac{x}{kT^{1/n}}\right)^n} \cdot (kT)^{\frac{1}{n}}\, d\frac{x}{(kT)^{\frac{1}{n}}}$$

$$= \underbrace{-\frac{\partial}{\partial(1/kT)} \ln (kT)^{\frac{1}{n}}}_{\frac{1}{n}kT} \underbrace{- \frac{\partial}{\partial(1/kT)} \underbrace{\ln \int e^{-b\left(\frac{x}{(kT)^{1/n}}\right)^n} \cdot d\left(\frac{x}{kT^{\frac{1}{n}}}\right)}_{\text{indep of } kT}}_{0}$$

§2 On Planck's Determination of the Elementary Quanta

The goal: TWO empirical constants in formulae for ρ_ν $\xrightarrow[\text{classical limit}]{\text{compare with}}$ Reduce to ONE ...other is Avogadro's number N. ... new h

Planck's empirical formula $$\boxed{\rho_\nu = \frac{\alpha \nu^3}{e^{\beta \nu / T} - 1}}$$ modern form $$\frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}$$

classical limit ν/T small

$$\rho_\nu = \frac{\alpha}{\beta} \nu^2 T$$

High ν/T limit Wien's Law

$$\boxed{\rho_\nu = \alpha \nu^3 e^{-\frac{\beta \nu}{T}}}$$

compare with $\rho_\nu = \frac{R}{N} \frac{8\pi\nu^2}{c^3} T$ from §1 $\leftarrow L$

$$\boxed{\frac{\alpha}{\beta} = \frac{R}{N} \frac{8\pi}{L^3}}$$

$\frac{k 8\pi}{c^3}$

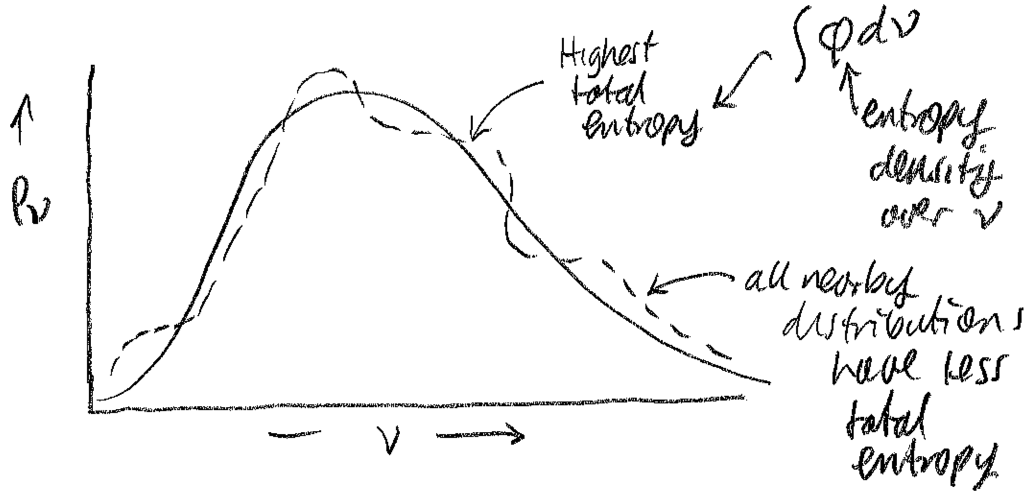
Results to be carried forward to section 4

§3 On the Entropy of Radiation

The goal : Thermodynamic law $\frac{\partial \varphi}{\partial \rho} = \frac{1}{T}$: Energy density $\underbrace{\rho}_{\text{observed!}}$ $\Longrightarrow$ entropy density $\underbrace{\varphi}_{\text{key to core argument}}$

Systems spontaneously move to states of higher entropy (if accessible) $\Longrightarrow$ Equilibrium state = accessible state of highest entropy

Equilibrium energy distribution in black body radiation?



$\Uparrow$

satisfies

$\Downarrow$

$$\boxed{\delta S = 0}$$

δ is arbitrary ("infinitesimal") variation in energy distribution ρ_ν

CONSTRAINED

by condition of constant total energy E

$$\boxed{\text{Solve: } \delta S = 0 \text{ subject to } E = \text{constant}}$$

$\Downarrow$ Introduce Lagrange multiplier λ

unconstrained variation
$$\delta\big(\underset{\int \varphi_\nu d\nu}{S} - \lambda(\underset{\int \rho_\nu d\nu}{E} - \text{constant})\big) = 0$$

vary with respect to λ
$$\underbrace{(E - \text{constant})}\delta\lambda = 0$$
$\therefore E = \text{constant}$

$$\therefore \delta\left(\int(\varphi_\nu - \lambda\rho_\nu)\, d\nu\right) = 0$$

vary with respect to ρ_ν

$$\therefore \int \underbrace{\left[\frac{\partial \varphi_\nu}{\partial \rho_\nu} - \lambda\right]}_{\text{must vanish}} \underbrace{\delta\rho_\nu}_{\text{arbitrary}}\, d\nu = 0$$

$$\boxed{\frac{\partial \varphi_\nu}{\partial \rho_\nu} = \lambda = \frac{1}{T}}$$

$\lambda = 1/T$ identified by imagining slow heating of frequency cut ν of black body radiation.

$$\text{Gain entropy} = \frac{\text{Gain Energy (heat)}}{T}$$

$$d\varphi_\nu = \frac{d\rho_\nu}{T}$$

Variation operator δ?

- **Informally**: infinitesimal change from $\rho_\nu(\nu)$ to $\rho_\nu(\nu) + \delta\rho_\nu(\nu)$. δ obeys calculus of derivative operators.

- **Precisely**: Define path in space of energy densities

$$\rho_\nu(\nu, \varepsilon) = \rho_\nu(\nu) + \varepsilon \underbrace{f(\nu)}_{\text{arbitrary}}$$

$$\delta = \frac{d}{d\varepsilon}\Big|_{\varepsilon=0}$$

$\delta[F(\rho_\nu)] = 0$ means $\frac{d}{d\varepsilon}\left[F(\rho_\nu(\nu,\varepsilon))\right] = 0$ for any $f(\nu)$

Constrained Optimization: method of Lagrange multipliers

optimize $L(x_1, \cdots, x_n)$ w.r.t. $x_1, \cdots, x_n$ subject to constraints $f_\alpha(x_1, \cdots, x_n) = 0$, $\alpha = 1, \cdots, \beta$	equivalent $\Longleftrightarrow$	Unconstrained optimization of $L^*(x_1, \cdots, x_n, \lambda_1, \cdots, \lambda_\beta)$ $= L(x_1, \cdots, x_n) + \sum_{\alpha=1}^{\beta} \lambda_\alpha f_\alpha(x_1, \cdots, x_n)$ w.r.t. $x_1, \cdots, x_n, \lambda_1, \cdots, \lambda_\beta$

Let $\dot{x}_1, \cdots \dot{x}_n, \dot{\lambda}_1, \cdots, \dot{\lambda}_\beta$ be values at which L^* optimized

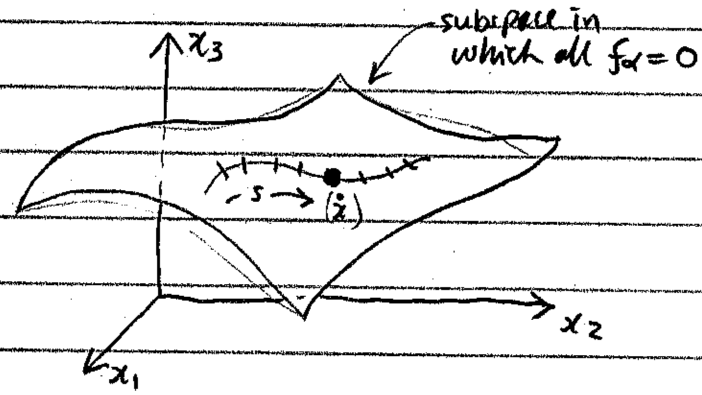
Now show this is a constrained optimum of $L(x_1, \cdots, x_n)$

I $\dot{x}_1, \cdots, \dot{x}_n$ lies in subspace in which all $f_\alpha = 0$

since at optimum

$$\frac{\partial L^*}{\partial \lambda_\alpha} = \underbrace{f_\alpha = 0}_{} \quad \text{all } \alpha$$

In practice, just stipulate directly ~~and infer back to $\dot{\lambda}_\alpha$~~



II If $(x_1(s), \cdots x_n(s))$ is any path through $(\dot{x}_1, \dot{x}_2, \cdots \dot{x}_n)$ in this subspace, then $\frac{dL}{ds} = 0$, $\frac{d^2L}{ds^2} \neq 0$ $\longrightarrow$ L is constrained optimization

$$0 = \frac{dL^*}{ds} = \frac{dL}{ds} + \sum_{\alpha=1}^{\beta} f_\alpha \underbrace{\frac{\partial \lambda_\alpha}{\partial s}}_{\sum_i \underbrace{\frac{\partial \lambda_\alpha}{\partial x_i}}_{0} \frac{dx_i}{ds}} = \frac{dL}{ds}$$

$$\therefore \frac{d^2L}{ds^2} = \frac{d^2L^*}{ds^2} \neq 0$$

§4 Limiting law for the entropy of monochromatic radiation at low radiation density

Wien's law for large ν/T $\quad \rho_\nu = \alpha \nu^3 e^{-\beta \nu/T}$

$\downarrow$ invert

$$\therefore \frac{1}{T} = -\frac{1}{\beta\nu} \cdot \ln \frac{\rho}{\alpha\nu^3}$$

$$\frac{\partial \varphi}{\partial \rho} = \frac{1}{T} \Rightarrow \varphi = \frac{-1}{\beta\nu} \int \ln \frac{\rho}{\alpha\nu^3} \cdot d\rho$$

$$= \frac{-1}{\beta\nu}\, \rho \left\{ \ln \frac{\rho}{\alpha\nu^3} - 1 \right\} + \overbrace{\text{const.}}^{\text{set to zero}}$$

Radiation of total energy E in volume v with frequency ν to $\nu + d\nu$ has entropy

$$S = v\, \varphi(\rho, \nu)\, d\nu = -\frac{E}{\beta\nu} \left\{ \ln \frac{E}{v\alpha\nu^3 d\nu} - 1 \right\}$$

Compare S, S_0 – same energy E, frequency ν – different volumes v, v_0] THIS ENTAILS THAT SYSTEMS HAVE DIFFENT TEMPERATURES!

$$S - S_0 = -\frac{E}{\beta\nu} \left\{ \ln \frac{E}{v\alpha\nu^3 d\nu} - \ln \frac{E}{v_0\alpha\nu^3 d\nu} \right\} = -\frac{E}{\beta\nu} \ln \left\{ \frac{v_0}{v} \right\}$$

$$\boxed{S - S_0 = \frac{E}{\beta\nu} \ln \left\{ \frac{v}{v_0} \right\}}$$

$$\downarrow$$

$$\left(k \frac{E}{h\nu} \right)$$

ν, T Fixes energy density ρ

so. same E, $d\nu$ different v

$\Downarrow$

different ρ

$\Downarrow$

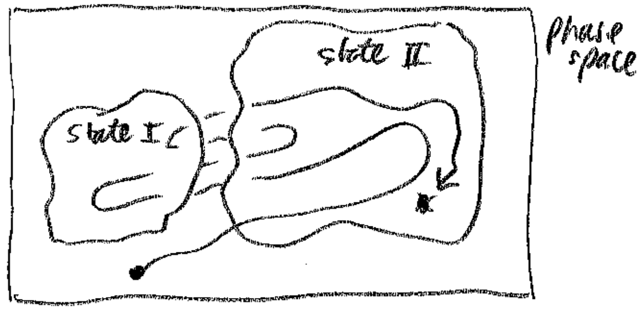
different T

§5 Molecular-Theoretical Investigation...

Demonstration of Boltzmann's Principle

$$S = \underset{\uparrow R/N}{k} \log \underset{\uparrow \text{probability}}{W}$$

Define probability as fractional occupation times (not equally probable cases)



less time → lower probability

more time → higher probability

system spontaneously moves to higher probability states

System spontaneously moves to higher entropy states S	+	System spontaneously moves to higher probability states W

And even that relation is a function at all!

⇓

$$\boxed{\begin{array}{c}S \text{ is a function of } W \\ S = \varphi(W)\end{array}}$$

JDN: Tacit assumptions !!!

(i) No other variables in φ

(ii) Entropy of state I above produced by fluctuation = Entropy of state I when it is in equilibrium

Not defined in ordinary thermodyn

Fix form of φ with constraint:

For two independent systems $\boxed{1}$ $\boxed{2}$

$$S = S_1 + S_2$$
$$W = W_1 \cdot W_2$$

- For such a φ, $S = \varphi(W) \implies -S = \varphi(A/W)$ for some constant A

since $\underbrace{0}_{\text{independent of } S} = S + (-S) = \varphi(\underbrace{W \cdot W^*}_{\text{independent of } W})$ where W^* is that W for which $-S = \varphi(W^*)$

independent of $S \implies$ independent of W ... say A hence $W \cdot W^* = A$

$W^* = A/W$

- $$\underbrace{\frac{S_2 - S_1}{W_2 - W_1}}_{\text{Take Limit } W_2 \to W_1} = \frac{\varphi(AW_2/W_1)}{W_2 - W_1} = \underbrace{\frac{\varphi(AW_2/W_1)}{W_2/W_1 - 1}}_{\text{must be independent of } W_2, W_1 \text{ as } W_2 \to W_1 \text{ ... call it } k} \cdot \frac{1}{W_1}$$

$$\frac{dS}{dW} = \frac{d\varphi}{dW} = \frac{k}{W}$$

Comparison with kinetic theory of gases $k = R/N$

e.g. analysis of ideal gas coming

$$\boxed{S - S_0 = k \log(W/W_0)}$$

Balazs Gyenis simplified proof

$$S(W_1 \cdot W_2) = S(W_1) + S(W_2) \Rightarrow \quad S(W) = k \log W$$

Assume
$S(w)$ differentiable
$0 < w \leq 1$

$$S(xy) = S(x) + S(y)$$

$$\xleftarrow{\partial/\partial x} \qquad \xrightarrow{\partial/\partial y}$$

$$y\, S'(xy) = S'(x) \qquad\qquad x\, S'(xy) = S'(y)$$

combine $\quad \dfrac{S'(x)}{y} = S'(xy) = \dfrac{S'(y)}{x}$

$\therefore \quad x\, S'(x) = y\, S'(y) =$ constant indep of x and y $=$ "k"

$\therefore \quad S'(x) = k/x$

$\quad S(x) = k \log x \; +$ constant

$S(W_1 \cdot W_2) = S(W_1) + S(W_2) \Rightarrow S = k\log W \underbrace{(+\text{ constant})}_{=0}$

Another proof that assumes differentiability only at ONE value of $W \neq 0$

$S(W_1 W_2) = S(W_1) + S(W_2)$ → Directly $S(1) = 0$ since, for $W_1 = W_2 = 1$, $S(1) = 2(S(1)) \therefore S(1) = 0$

$\therefore S(W_1 W_2) - S(W_1) = S(W_2)$

set $W_2 = 1 + \varepsilon/W_1$

$\therefore S(W_1 + \varepsilon) - S(W_1) = S(1 + \varepsilon/W_1)$

$$\therefore \underbrace{\frac{S(W_1+\varepsilon) - S(W_1)}{\varepsilon}}_{\text{If there exists any value of } W_1 \neq 0 \text{ at which } S \text{ is differentiable, then } \lim_{\varepsilon \to 0} \text{ of this qty exists}} = \frac{1}{W_1} \cdot \underbrace{\frac{S(1 + \varepsilon/W_1)}{\varepsilon/W_1}}_{\lim_{\varepsilon\to 0} \text{ of this qty must exist as well}}$$

If there exists any value of $W_1 \neq 0$ at which S is differentiable, then $\lim_{\varepsilon \to 0}$ of this qty exists $\Longrightarrow$ $\lim_{\varepsilon \to 0}$ of this qty must exist as well ... but this limit is indep. of W_1 call it "k" $\Rightarrow$ $\lim_{\varepsilon \to 0}$ of LHS exists for all other values $W_1 \neq 0$

If this limit exists, then $S(1) = 0$

i.e. For all $W \neq 0$ $\quad \frac{dS}{dW} = \frac{k}{W} \Rightarrow S = k\log W + \underbrace{\text{constant}}_{=0 \text{ since } S(1)=0}$

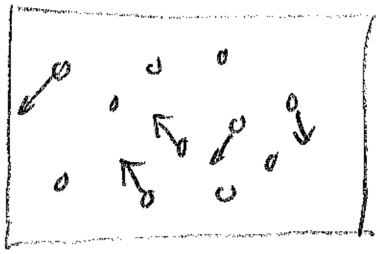
... and $S(W)$ is either everywhere differentiable for $W \neq 0$ or nowhere differentiable

[Conjecture: Rule out nowhere differentiable case by requiring that S is strictly increasing]

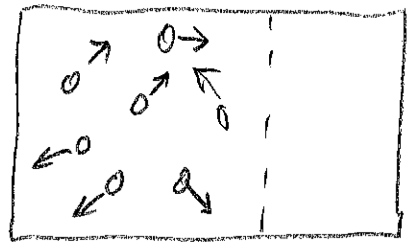
Apply Boltzmann's Principle to system of n independently moving components (localized points)
e.g. ideal gas
dilute solute

System occupies volume v_0

System occupies volume v



ordinary time development



Entropy S_0

Entropy S

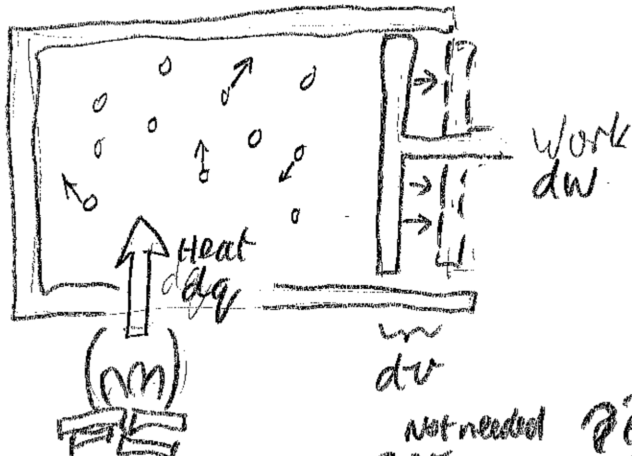
Probability W of transition $= \left(\frac{v}{v_0}\right)^n$ ← Prob v/v_0 for each component. n times, independently
A hugely unlikely fluctuation

$$\boxed{S - S_0 = \frac{R}{N} \ln W = \frac{R}{N} \ln\left(\frac{v}{v_0}\right)^n = R\underbrace{\frac{n}{N}}_{kn} \ln v/v_0}$$

$$\boxed{S - S_0 = R\,\frac{n}{N}\ln v/v_0 \quad \Longleftrightarrow \quad \text{Ideal gas law } \; pv = \frac{n}{N}RT}$$

↑ Pressure

Consider reversible isothermal expansion



Reversible

⇓

System always at equilibrium

⇓

$$dq = TdS$$
$$dW = pdV$$

Not needed ?? ΔE assumes two states have same energy E

Since T fixed, internal energy stays constant at $n(\frac{1}{2}kT)$ (equipartition)

$$\Longrightarrow \underbrace{0 = dE = dq - dW = TdS - pdV}$$

Einstein writes this as

$$-d(\underbrace{E - TS}_{\text{Free energy}}) = pdV = TdS \quad \text{... WHY?}$$

$$S = \frac{R}{N}\,n\ln v + \text{const.}$$
$$\Downarrow$$
$$dS = \frac{R}{N}\cdot\frac{n}{v}\,dv$$

$$\therefore\; pdv = TdS = \frac{RT}{N}\cdot\frac{n}{v}\,dv$$

$$\therefore\; \boxed{pv = n\frac{R}{N}T}$$

and conversely

... but does not work for radiation. See later.

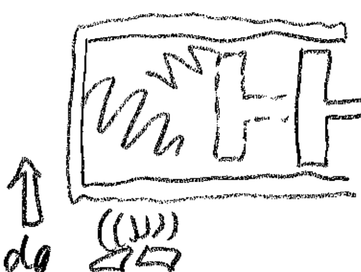
Recover "ideal gas" law for light quanta? NO

YES! see Later

$$S - S_0 = k \underbrace{\frac{E}{h\nu}}_{n} \ln \frac{v}{v_0} = k n \ln v/v_0 \quad \Longrightarrow \quad pv = nkT \;\; \& \;\; E = 3nkT$$

for radiation same frequency $\nu \to \nu + d\nu$
& same energy E

Reversible expansion.



work dw $\quad 0 = dE = \overbrace{T\,\underbrace{dS}_{kn\,dv/v}}^{dq_{rev}} - p\,dv$

dq

add just enough heat to keep internal energy E constant

i.e. $T \frac{kn}{v} dv = p\,dv$

$\therefore \boxed{pv = nkT}$ ... ideal gas law for quanta!!

For isotropic radiation $p = \rho/3$

$\therefore E = \rho v = 3pv = 3nkT$

FALLACY: In expansion, frequency ν changes according to Wien displacement law – reflection off moving surface of piston
Hence $S - S_0 = kn \ln v/v_0$ inapplicable.
must use more detailed expression, which includes ν dependence!

... Also, how do we add heat without disturbing presence of just one frequency?

Compare with §2 of Brownian motion paper
same argument — rendered more informally!

System of n spatially localized components
moving independently in volume v

Light Quantum paper

Boltzmann principle
$S = k \ln W$

$S = k \ln v$ + const.

Reversible isothermal expansion
$-dF = p\,dV = T\,dS$

? ?

same!

Free energy
$F = \bar{E} - TS$
$= -kT \ln \int \exp\left(\frac{-E}{kT}\right) dp_1 \ldots dp_\ell$

Brownian motion paper

$F = -kT\left[\ln J + n \ln v\right]$

Pressure
$p = -\frac{\partial F}{\partial V}\Big|_T$

Ideal gas law $pv = R\frac{n}{N}T$

Recover light Quantum

Entropy of volume of radiation v in high frequency range ν to $\nu + d\nu$

$$S - S_0 = \frac{E}{\beta\nu} \ln\left(\frac{v}{v_0}\right)$$

$$= \frac{R}{N} \ln\left(\frac{v}{v_0}\right)^{\frac{N}{R}\frac{E}{\beta\nu}}$$

+

compare to Boltzmann's Principle

$$S - S_0 = \frac{R}{N} \ln W$$

$\Downarrow$

Probability radiation spontaneously all found in volume v

$$W = \left(v/v_0\right)^{\frac{N}{R}\cdot\frac{E}{\beta\nu}}$$

$\Downarrow$

"... monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as if it consisted of mutually independent energy quanta of magnitude $\underbrace{\frac{R\beta\nu}{N}}_{h\nu}$.."

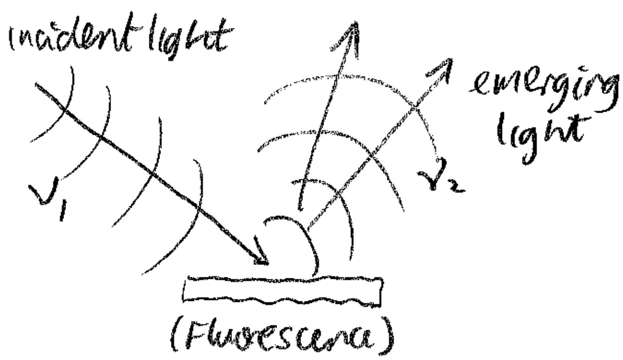
AE: mean energy over all ν of quanta is $3\frac{R}{N}T$

JDN: significance?

§7-9 Evidence for Light Quantum in Processes of Production & Transformation of Light

Theme:	Wave theory:	Quantum theory
	Causal powers of light depend on INTENSITY	Causal powers of light depend on FREQUENCY
	e.g. low intensity ⇓ no effect	e.g. low frequency ⇓ no effect
Display experiments	Fails	obtains

§7 Photoluminescence:

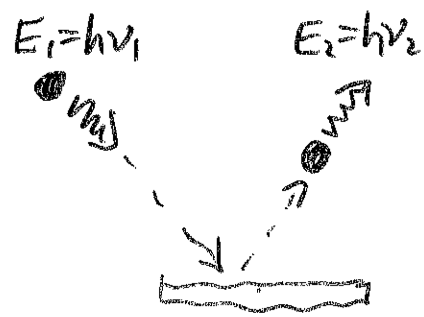


Wave theory: no explanation

⇓

$$\boxed{\text{Stokes law} \quad \nu_2 \leq \nu_1}$$

⇑ $E = h\nu$

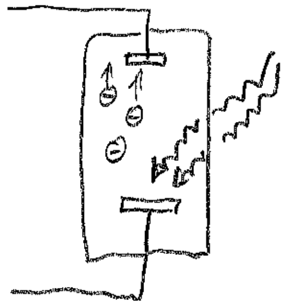


$$\boxed{\text{conservation of energy} \quad E_2 \leq E_1}$$

NB: Expect exceptions outside Wien regime for light

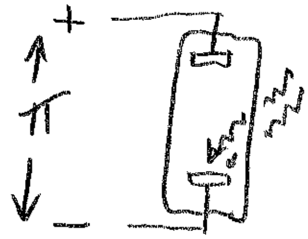
98 Photoelectric effect

The effect



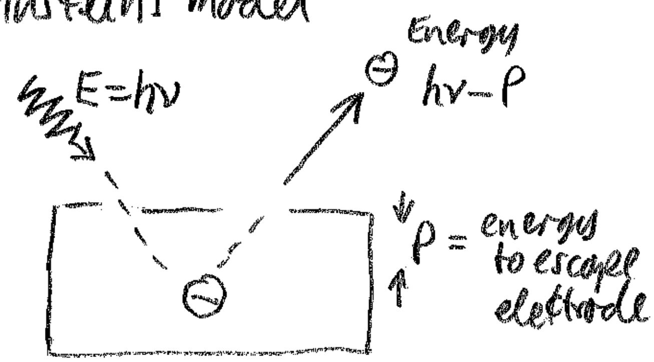
Light falls on electrode
⇓
Electrons knocked out
⇓
current can flow.

Determine energy of emitted electrons by applying opposing voltage Π



Stopping voltage Π = minimum voltage to stop current.

Einstein's model



Einstein predicts

single charge

$$\Pi \varepsilon = h\nu - P$$

$$\Pi E = Nh\nu - \underbrace{NP}_{\text{"P"}}$$

mole of charge

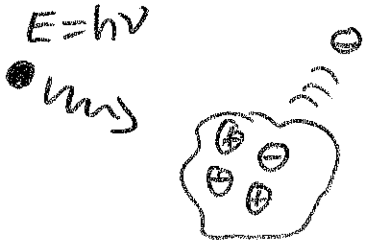
Effect depends linearly on frequency & not on intensity

Nobel Prize!

§9 Ionization of Gases by UV light

model

$E = h\nu$



atom

must have

$$Nh\nu \geq J$$

$\uparrow$ ionization energy for one gram mole

> NB: Bohr model is 1913!